

Asset Allocation: Risk Models for Alternative Investments

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Often, the lack of mark-to-market data lures investors into the misconception that alternative asset classes and strategies represent somewhat of a “free lunch.” This article proposes solutions to measuring mark-to-market risk in alternative and illiquid investments. The authors describe how to estimate risk factor exposures when the available asset return series may be smoothed (owing to the difficulty of obtaining market-based valuations). They show that alternative investments are exposed to many of the same risk factors that drive stock and bond returns.

Investors have long recognized that asset-class returns are driven by a common set of risk factors. Asset allocators often use the risk factor approach to improve portfolio diversification and to translate macroeconomic views into expected asset returns. In practice, implementing a risk factor approach to asset allocation requires mapping asset classes to their underlying factor exposures, which can be challenging, especially for asset classes for which the available historical data are limited or biased.

In this article, we propose solutions to measuring mark-to-market risk in alternative and illiquid investments. We describe how to estimate risk factor exposures when the available asset return series may be smoothed (owing to the difficulty of obtaining market-based valuations). We show that alternative investments are exposed to many of the same risk factors that drive stock and bond returns.

Our approach has profound implications for risk estimation in an asset allocation context. For example, **Figure 1** shows the difference between adjusted and reported (from index returns) volatilities for several alternative investments, as well as for public markets (equities and bonds). It also shows a measure of autocorrelation, which highlights how return smoothing contributes to the misestimation of volatility. The bottom line is that alternative investments are much more volatile—on a mark-to-market basis—than their reported

index returns would suggest. This bias tends to be more pronounced for indices that are smoothed.

In this article, we describe the methodology used to arrive at these adjustments and include other key risk measures relevant to asset allocation. We recognize that there already is a significant body of literature that attempts to estimate risk factor exposures for various individual alternative investments and strategies. However, little research has been done to estimate the risk factor exposures across all alternatives within an internally consistent, unified risk factor framework. Given increased allocations to alternative investments in institutional investors' portfolios, we see an urgent need to develop a consistent approach that directly integrates the risks of alternative assets with the rest of investors' portfolios.

Measuring Risk across Alternative Investments

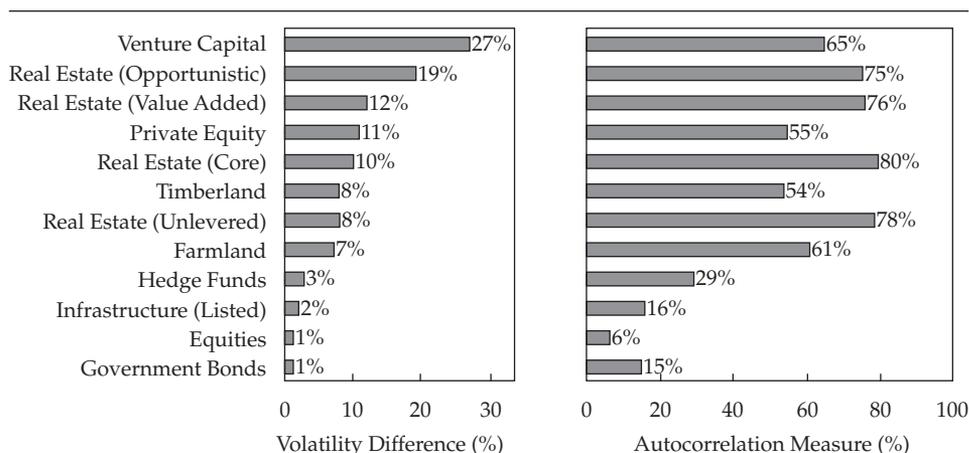
We classify alternative investments broadly into three groups:

1. private equity and venture capital;
2. real assets—that is, real estate, infrastructure, farmland, timberland, and natural resources; and
3. hedge funds and exotic beta strategies (momentum, carry, value, volatility, etc.).

Often, the lack of mark-to-market data lures investors into the misconception that these asset classes and strategies represent somewhat of a “free lunch.” Their relatively high returns appear to come with low risk and significant diversification with respect to traditional asset classes in normal times. This misconception arises because return indices for privately held assets often are artificially smoothed, which biases both volatility and correlation estimates downward.

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Figure 1. Volatility Difference (Downward Bias in Standard Deviation Due to Smoothed Returns) and Autocorrelation Measure by Asset Class



Notes: The analysis is based on quarterly data from December 1991 through December 2012, except for timberland and farmland data, which are annualized owing to unreliable quarterly data, and hedge fund data, which are monthly and are from January 1995 through December 2012. Indices used for asset classes and their underlying risk factors are listed in Appendix B. The autocorrelation measure is the sum of the coefficients on significant lags using the methodology outlined in Appendix A. The number of significant lags (“Q” from Equation 3) is two years for timberland and farmland, five quarters for venture capital and private equity, six quarters for all real estate asset classes, and one quarter for listed infrastructure, stocks, and bonds.

Sources: PIMCO; Cambridge Associates; NCREIF (National Council of Real Estate Investment Fiduciaries); Bloomberg.

To address this problem, risk models for private asset classes should rely on public proxies or publicly traded equivalents. Also, the statistical methods used to estimate correlation and volatilities for these assets must be adjusted to reflect the nature of the reporting biases in the illiquid return series.¹ To do so, investors must identify the systematic return drivers that affect each of their alternative investments. If the risk model fails to capture the systematic risk factor exposures, diversification benefits may be overestimated.

Combining Fundamental Valuation with Empirical Analysis

An assessment of which factors to include requires the use of econometric methods as well as judgment. Alternative assets’ risk factor exposures have been the subject of extensive research.

Private Equity and Venture Capital. Private equity and venture capital have often been studied in comparison with public market equity investments. A frequently asked question is whether private equity tends to outperform public equity markets. Recently, new, more comprehensive databases have allowed for in-depth analysis of the systematic risk factor exposures of private equity. Franzoni, Nowak, and Phalippou (2012) made an important contribution to the literature on private

equity by estimating a four-factor model for private equity and venture capital returns based on an exhaustive set of realized returns of individual private equity investments. The authors found that “private equity suffers from significant exposure to the same liquidity risk factor as public equity and other alternative asset classes” (p. 2341).

Real Estate. Academics and practitioners have studied real estate using different approaches and have often arrived at conflicting conclusions. Fisher, Geltner, and Webb (1994) and Fisher and Geltner (2000) extensively studied the volatility and return characteristics of real estate. The work of Pedersen, He, Tiwari, and Hoffmann (2012) on risk factor models for real estate provided the theoretical and practical foundations behind the framework suggested in this section, which we applied across alternative asset classes. The key feature of their approach is to incorporate fundamental valuation principles into the process of deciding which risk factor to use. They showed that the risk characteristics of private and public real estate are more similar than many believe, once risk factors have been carefully selected and smoothing, liquidity, and leverage effects have been accounted for. This result squares with economic intuition because the underlying investments are essentially the same.

Hedge Funds. Hedge funds are probably the most extensively studied asset class of our three groups, yet findings in this area remain controversial, especially regarding whether hedge funds can be cloned (Laise 2009). However, there is a fair degree of consensus that market-related factors explain a large proportion of hedge fund volatility. Some of the main contributions to this literature were made by Fung and Hsieh (1999, 2001, 2002, 2004a, 2004b); Fung, Hsieh, Naik, and Ramadorai (2008); Jaeger and Wagner (2005); Agarwal and Naik (2004); and Hasanhodzic and Lo (2007). Hedge fund returns also suffer from the smoothing bias, owing to illiquidity and reporting issues (Getmansky, Lo, and Makarov 2004).

Our models offer a practical interpretation of the existing literature for each asset class. Importantly, we use a uniform set of factors and a similar econometric approach across all asset classes.

Econometric Modeling of Alternative Asset Classes

A “kitchen sink” regression approach—which starts from a very expansive set of risk factors, however sophisticated it may be—tends to isolate factors that improve the fit in sample but produces exposures without clear economic interpretation. Often, the associated risk models tend to perform poorly out of sample.² For this reason, our approach to assigning risk factor exposures to alternative asset classes consists of two steps:

- First, we use economic intuition to narrow down the set of factors that should be relevant for a particular alternative asset class or strategy. This process relies on basic valuation principles and knowledge of the underlying investments.
- Second, we use econometric techniques to estimate exposures to each factor on the basis of historical returns. To adjust for the smoothing effect, our model assumes that observed index returns represent a “moving average” of the current and past “true” investment returns. Dimson (1979) and Scholes and Williams (1977) presented some of the theoretical foundations for this approach, and we describe our model in detail in Appendix A. For related but non-factor-based methods used to unsmooth data, see Geltner (1993); Getmansky et al. (2004); and Gallais-Hamonno and Nguyen-Thi-Thanh (2007).

Risk Factors for Private Equity, Venture Capital, and Real Assets

This two-step process means that before we embark on our empirical analysis, we must identify the most important set of risk factors for each asset

class. (Hedge funds require a separate treatment, as explained in the following section.)

If we accept that investors value alternative assets as discounted cash flow streams, we should expect their volatility to be driven by the same factors that drive expected growth and discount factors for stocks and bonds (of course, there is always the possibility of idiosyncratic variance or missing factors). For assets with stable and less cyclical cash flow dynamics, valuation changes should be dominated by changes in interest rates—just as interest rates drive most of the volatility for bonds. In contrast, valuations for more speculative and highly cyclical investments should be driven by changes in the risk premiums that investors require for risky assets; therefore, such investments should exhibit equity-like characteristics.

Table 1 shows the risk factor exposures we used to model private equity, venture capital, real assets, and hedge funds. The table also reports univariate regression equity betas,³ as well as risk exposures for equities and bonds, for comparison purposes.

To address the smoothing bias, our model uses transformed risk factor returns that account for the lag structure of the index. We have kept the list of factors parsimonious and consistent with those used for stocks and bonds. Reported betas represent the sum of the current and lagged betas, based on the model discussed by Getmansky et al. (2004) to address liquidity biases, extended to a multifactor framework.

Our analysis is based on quarterly data from December 1991 through December 2012, except for those for timberland and farmland, which are annualized owing to unreliable quarterly data, and for hedge funds, for which we used monthly data from January 1995 through December 2012. A convenient feature of our approach is that to the extent the betas are stationary, annual and quarterly data can be modeled at a higher frequency—for example, monthly—once assets have been mapped to risk factors for which higher-frequency data are available. This process provides an efficient way to combine data of various frequencies into a common correlation matrix, and it can be used to backfill missing historical data (Page 2013).

The *t*-statistics, which we report below each coefficient, are based on the Newey–West (1987) approach, which controls for autocorrelation biases. We show the data sources for risk factor and asset-class index returns in Appendix B.

Private equity, venture capital, and real assets are exposed to the following risk factors.

Equity Beta. Equity beta represents most of the mark-to-market risk across alternatives because equity market returns reflect changes in how

Table 1. Risk Factor Exposures and *t*-Statistics for Alternative Investments and Sample Public Markets

Asset Class	US Equity	EM Equity	Size	Value	Liquidity (P&S)	Industry Factors	Nominal Duration	Real Duration	IG Corp. Spread	HY Corp. Spread	EM Spread	Commodity	Volatility	Momentum	Equity Beta (Univ.)
Private equity	0.8		-0.9	-1.4	0.2					1.2					0.9
	6.5		-1.4	-3.3	1.9					1.5					12.1
Venture capital	1.4		-3.9	-2.9	0.2										1.6
	5.4		-2.9	0.6	0.6										5.8
Infrastructure	0.5		0.7	1.1	0.2	1.1	0.6	0.6	7.3						0.6
	3.8		1.1	1.4	1.4		0.4	0.4	2.6						5.8
Farmland	0.1					0.9	9.7								0.0
	1.0						2.3								-0.2
Timberland	0.3					2.0	9.1	9.1	2.6						0.2
	1.4						1.2	1.2	0.4						1.1
Real estate (unlevered)	0.3				0.4	0.5	1.2	1.2	1.7						0.3
	5.2				5.4		1.0	1.0	1.3						4.9
Real estate (core)	0.4				0.5	0.6	2.8	2.8	2.7						0.5
	5.2				4.9		1.8	1.8	1.4						5.7
Real estate (value added)	0.5				0.6	1.2	2.4	2.4	3.0						0.6
	4.4				4.3		1.1	1.1	1.1						5.1
Real estate (opportunistic)	0.5				0.8	1.7			11.8						0.8
	3.6				4.7				3.7						5.3
Hedge funds	0.3	0.1	-0.9	-0.2	0.03		-0.6		2.5	-0.8	0.5	0.03	0.1	0.04	0.4
	8.4	6.4	-6.5	-1.7	1.5		-1.5		3.0	-3.5	3.6	1.8	2.1	2.0	16.8
Equities	1.0		0.8	0.3											1.0
	19.3		1.5	0.8											43.5
Gov't. bonds							5.1								-0.2
							26.0								-5.3

Notes: Numbers in bold are coefficients from regressions estimated on adjusted risk factor returns based on the lag structure in index data (see Appendix A for methodology), whereas numbers in italics are Newey–West *t*-statistics. The indices used for asset classes and their underlying risk factors are listed in Appendix B. P&S = Pastor and Stambaugh; IG = investment-grade; HY = high-yield; Univ. = univariate.

Sources: PIMCO; Cambridge Associates; NCREIF; Bloomberg.

investors value and discount cash flow streams at a broad level—as evidenced by the financial crisis of 2007–2009.

As for corporate earnings, cash flows for private assets are linked to general economic growth. Company profitability and earnings growth can be expected to be high during expansions and low during recessions, irrespective of whether a specific company is traded privately or publicly. The same logic applies to real estate and infrastructure investments, whose cash flows—and, therefore, market values—vary with the level of economic activity.⁴

Other Equity Factor Betas. Other equity factor betas help better capture asset-class-specific risk exposures. Our models incorporate style (size and value) and industry-specific equity factors to account for exposures that may be independent of broad equity beta. We source the returns for these factors from Barra (Menchero, Morozov, and Shepard 2010). Note that by virtue of the Barra methodology, these factors' returns are expressed as “net of the equity world factor”; hence, by construction, they are almost perfectly independent. For simplicity, Table 1 reports the sum of industry betas in the “Industry Factors” column, but each industry is treated as a separate variable. **Table 2** shows the details of which industry factor exposures we used for each asset class. Both venture capital and private equity typically have

small and growth biases, as measured by their sensitivities to these Barra factor returns. Real estate is exposed to the Barra real estate factor.

Credit Spread Duration. Credit spread duration captures bond-like cash flow risk and financing effects. Whereas equity returns capture some of the common variation in discount rates across alternative asset classes, credit spreads may play a distinct role in shaping the returns for some alternative investments, such as real estate and infrastructure. Owing to the nature of their bond-like cash flows, the pricing of some real assets may be more closely correlated with bond spreads than with equity valuations. In other words, credit spreads are a key component of the discount rate applied by investors to the cash flow streams of real asset investments because they are viewed in part as substitutes for bonds. In addition, most private equity, real estate, and infrastructure portfolios are exposed to financing or refinancing risks. Owing to this exposure, anticipated returns can be particularly vulnerable to changes in the costs and availability of debt financing, both of which change with credit spreads.

Real Interest Rate Duration. Real interest rate duration represents the inflation-hedging characteristics of certain alternative asset classes.

Table 2. Industry Exposures and *t*-Statistics, December 1991–December 2012

	Real Estate	Food Retail	Food Products	Paper	Utilities	Transportation
Infrastructure					0.5	0.6
					<i>2.2</i>	<i>2.0</i>
REITs	1.4	0.3	-0.6			
	<i>4.7</i>	<i>0.7</i>	<i>-1.5</i>			
Farmland	1.3			1.9		
	<i>4.6</i>			<i>1.9</i>		
Timberland	0.2					
	<i>0.4</i>					
NCREIF property	0.5					
	<i>3.6</i>					
Real estate (core)	0.6					
	<i>3.5</i>					
Real estate (value added)	1.2					
	<i>4.8</i>					
Real estate (opportunistic)	1.7					
	<i>5.6</i>					

Notes: Numbers in bold are coefficients from regressions estimated on adjusted risk factor returns based on the lag structure in index data (see Appendix A for methodology). Numbers in italics are the Newey–West *t*-statistics. The analysis is based on quarterly data, except for timberland and farmland data, which are annualized owing to unreliable quarterly data.

Sources: PIMCO; NCREIF; Bloomberg.

Real estate investments provide *real* cash flows that are broadly insensitive to the level of inflation and nominal cash flows that track inflation over the medium to long term. Rent payments can, for example, be modeled as cash flows that are similar to coupon payments on an inflation-indexed bond because rent changes tend to reflect the general level of inflation.

Similarly, managers of infrastructure investments (such as toll roads and electricity producers) often have opportunities to adjust prices in response to inflation, at least partially. In the case of real estate, rents are a direct and significant component of inflation. Therefore, real estate and infrastructure investments should be mostly exposed to changes in real interest rates and less sensitive to changes in nominal rates. (In certain cases, where inflation pass-through is limited, it is appropriate to also consider assigning some nominal duration in the risk factor model.)

Liquidity Beta. Liquidity beta represents an important, yet often overlooked, component of the investment risk of most alternative asset classes (Page, Simonian, and He 2011). Indeed, decisions to allocate to private and illiquid asset classes are often made without serious consideration of their exposure to liquidity risk. To capture illiquid asset returns' exposure to fluctuations in liquidity, we included the liquidity factor of Pastor and Stambaugh (2003) in our models for real estate, private equity, and infrastructure. The Pastor–Stambaugh factor captures excess returns on stocks with large exposures to changes in aggregate liquidity. Pastor and Stambaugh constructed their liquidity measure for each stock by estimating the return reversal effect associated with a given order flow (volume); the idea is that lower-liquidity stocks will experience higher return reversals following periods of high volume. Then, they aggregated these liquidity estimates to form a market-wide liquidity measure at each point in time. The return to the liquidity risk factor in a given period is defined by the returns of a long–short portfolio of stocks that have been sorted according to their sensitivity to changes in market liquidity (“liquidity betas”). This methodology is similar to that used to derive the Fama–French (1992) factors.

Recent academic research by Franzoni et al. (2012) confirmed that realized private equity returns are affected by their significant exposure to the Pastor–Stambaugh liquidity factor. The authors described the economic channel that links private equity to public market liquidity. They explained how changes in illiquidity affect returns through availability and costs of financing for private equity deals:

Due to their high leverage, private equity investments are sensitive to the capital constraints faced by the providers of debt to private equity, who are primarily banks and hedge funds. Therefore, periods of low market liquidity are likely to coincide with periods when private equity managers may find it difficult to finance their investments, which in turn translate into lower returns for this asset class. (Franzoni et al. 2012, p. 2343)

The effects of liquidity are not just confined to private real assets. Liquidity conditions should affect the viability of all levered investments and should drive correlation across assets, especially during periods of stress. Modeling a common liquidity beta across alternative assets should help capture this effect.

Note, however, that liquidity exposures are also embedded in spreads; hence, the coefficients in Table 1 must be interpreted as exposures to “incremental systemic liquidity,” net of the liquidity effect embedded in other factors. Whereas the Pastor–Stambaugh factor relates to transactional liquidity, the investment-grade and high-yield spread factors, in particular, can be linked to the type of funding liquidity discussed in Brunnermeier and Pedersen (2009). In this context, 7 of the 10 alternative asset classes have significant exposure to *some* form of liquidity risk. The exposure of farmland and timberland is ambiguous because of data limitations, and to a certain extent, the same could be said of venture capital. Some asset classes are exposed to both liquidity proxies. As expected, the two reference liquid markets (equities and government bonds) have no liquidity factor betas. In this context, our framework provides hints as to how the two theories of liquidity—transactional and funding—can be integrated into an asset allocation framework. Choosing to allocate to alternative assets means, in part, choosing a desired level of liquidity risk.

Risk Factors for Hedge Funds

To analyze hedge fund style index returns, we expanded our list of risk factors. The expanded list consists of both conventional risk factors—such as US equity, emerging market (EM) equity, commodity, duration, spread exposures (investment grade, high yield, EM)—and more specialized “alternative beta” risk factors, such as foreign exchange carry, exposure to volatility, and momentum (trend following).

The set of risk factor exposures we suggest for a broad hedge fund index is shown in Table 1.⁵ The estimated exposures are based on the same regression approach that we described for private

and real assets. We used this approach because hedge funds have strong serial correlation in their monthly reported returns, which indicates illiquidity and smoothing of returns (see Getmansky et al. 2004). As before, the *t*-statistics are estimated using the Newey–West (1987) approach owing to serial correlation.⁶

As with other alternative assets, equity beta plays an important role in hedge fund risk. The motivation for including hedge fund allocations in multiasset portfolios is generally to diversify and limit exposure to equity risk. Therefore, it is especially important to estimate the relationship between hedge fund returns and the equity factor and to evaluate how robust the relationship is likely to be in stressed markets. Most hedge fund styles tend to have significant exposure to equity risk (directly or indirectly), which may be inconsequential until a crisis occurs.

Putting It All Together: Risk Estimates

Table 3 compares volatilities based on published index returns with estimated (“unsmoothed”) index return volatilities calculated using our model for all alternative investments discussed in this article. Estimated volatility can be decomposed into two components:

- *Factor-based volatility.* To estimate volatility from risk factors for a given asset class, we used the standard portfolio risk formula, but we replaced weights, volatilities, and correlations with risk factor exposures, risk factor volatilities, and risk factor correlations, respectively.
- *Non-factor-based volatility (idiosyncratic risk).* We added idiosyncratic volatility so that total volatility matches the unsmoothed index volatility. Idiosyncratic volatility can come from security selection, factor timing, and a variety of other nonsystematic, non-factor-based risk exposures, and it is assumed to have zero correlation with factor-based volatility. The results reported in Table 3 are the *contributions* from idiosyncratic volatility.

This analysis reveals, as expected, that volatilities calculated directly from index returns are much lower than those calculated using our unsmoothed estimates. Unsmoothing the return data increases volatility across *all* asset classes. For certain asset classes, the difference is material. In general, private equity and real assets are more sensitive to the smoothing bias than hedge funds are. Venture capital, real estate, and private equity are particularly sensitive asset classes.

Table 3 also compares equity correlations and betas from published returns with estimates from our models. Because we used common risk factors

Table 3. Volatilities, Correlations, and Equity Betas: Reported vs. Adjusted, December 1991–December 2012

	Volatility				<i>F</i> -Test	Equity Correlation		Equity Beta	
	Reported	Adjusted	Factors	Idiosyncratic		Reported	Adjusted	Reported	Adjusted
Private equity	11%	22%	17%	5%	0%	75%	75%	0.5	1.0
Venture capital	25	52	26	26	0	41	45	0.6	1.4
Infrastructure (listed)	15	17	12	6	20	56	56	0.5	0.6
Farmland	7	14	12	2	1	–13	1	–0.1	0.0
Timberland	9	17	11	6	1	11	18	0.1	0.2
Real estate (unlevered)	5	13	9	4	0	13	52	0.0	0.4
Real estate (core)	6	16	11	5	0	12	47	0.0	0.5
Real estate (value added)	9	21	14	7	0	16	49	0.1	0.6
Real estate (opportunistic)	12	31	22	9	0	31	47	0.2	0.9
Hedge fund index	7	9	8	1	0	76	74	0.4	0.4
Equities	18	19	19	1	67	99	95	1.0	1.0
Bonds	5	6	6	0	20	–62	–56	–0.2	–0.2

Note: The analysis is based on quarterly data, except for timberland and farmland data, which are annualized owing to unreliable quarterly data.

Sources: PIMCO; Cambridge Associates; Dow Jones Credit Suisse; NCREIF; Bloomberg.

for equities—including direct equity beta—it is not surprising that our models generate higher (and, we argue, more realistic) equity correlations. We believe these results provide evidence that our models better account for mark-to-market risk.

The table also shows the results of an *F*-test used to measure whether the differences between reported and adjusted volatilities are statistically significant (i.e., before versus after adjusting for serial correlation). A result of 0% indicates, with near certainty, that we cannot reject the hypothesis that the distributions are different. Only listed infrastructure, equities, and bonds fail this test, which we expected because these are public market indices.⁷

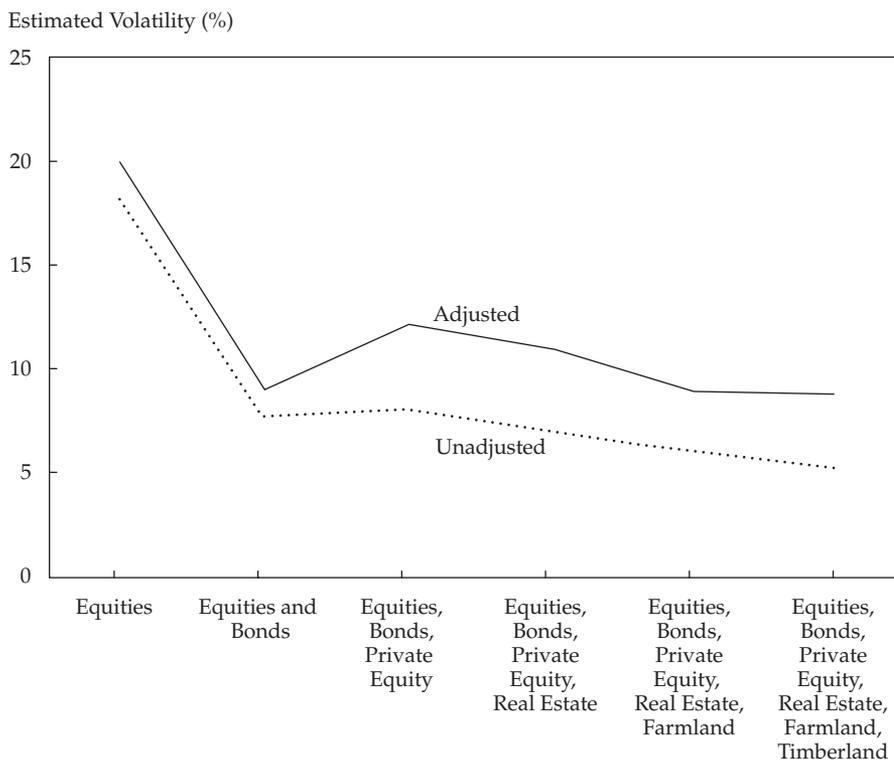
The Diversification Power of Alternatives: As Good as It Seems?

Our results have implications for asset allocation because reported returns may overstate the oft-touted diversification benefits of alternative investments. **Figure 2** illustrates the illiquidity effect on portfolio risk. We created equally weighted portfolios with increasing numbers of asset classes, starting with stocks, then including bonds, and incrementally adding the following key illiquid assets:

private equity, real estate, farmland, and timberland. All portfolios are equally weighted (“1/*n*”) portfolios. The curvature of the lines in the graph shows the effect of diversification.

Starting from the left, the volatility estimates for stocks and bonds are quite similar. The effect of unsmoothing index data for liquid markets is unlikely to be statistically significant. Next, private equity adds risk to a portfolio of stocks and bonds. This result is expected because private equity’s volatility is more than twice as high as that of an equally weighted portfolio of stocks and bonds. Crucially, as we add illiquid assets, the two lines start to diverge. Our estimate of unsmoothed portfolio volatility for the six-asset portfolio remains relatively high, at 8.8%, compared with 5.3% for the estimate based on reported index return data. This significant difference is due to our volatility adjustments and the increase in implied correlations among all assets in the portfolio (due to the use of common mark-to-market risk factors). A volatility of 8.8% is not too far from that of the initial stock and bond portfolio (9.0%). Nonetheless, our approach should *not necessarily* lead investors to avoid illiquid assets; investors should simply require a higher rate of return than they would otherwise.

Figure 2. Portfolio Volatility Estimates: Reported vs. Adjusted, December 1991–December 2012



Note: The analysis is based on quarterly data, except for timberland and farmland data, which are annualized owing to unreliable quarterly data.

Sources: PIMCO; Cambridge Associates; Dow Jones Credit Suisse; NCREIF; Bloomberg.

Whereas traditional risk models for alternative assets typically lead to corner solutions and a false impression that these assets represent a free lunch, our approach produces a more reasonable representation of the risk–return trade-offs involved in this important asset allocation decision.

Takeaways

Mean–variance optimization based on smoothed return indices often suggests extremely high optimal allocations to alternative assets owing to their low realized volatility and low correlation vis-à-vis publicly traded investments in liquid markets. However, our risk factor framework reveals that alternative assets have significant exposure to the same risk factors that drive stock and bond volatility. Returns on alternative assets depend on changes in interest rates, as well as how investors value risky cash flows, as reflected in equity market valuations and credit spreads. Liquidity and other specialized factors also play a role. In addition to higher volatility, expected drawdowns, and tail risk exposures, the approach based on risk factors typically generates higher correlations between alternative investments and their public market counterparts, especially when their equity betas are high.

When our models are applied to portfolio optimization problems, the relative attractiveness of alternative assets may be reduced. This result lends credibility to our approach. At equilibrium, there should not be any systematic free lunches and investors' optimal portfolios should not look much different from the total market portfolio. Of course, markets constantly deviate from equilibrium, but nonetheless, portfolio optimization results should reveal that our approach is much more in line with financial theory.

Overall, we recognize that our risk factor models can go only so far in describing the risk of alternative assets, but our approach should perform better (in the sense of giving a more accurate picture of potential drawdowns and volatility) than simply using artificially smoothed index returns. Importantly, it provides a coherent framework to aggregate risk exposures across public markets and alternative investments.

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This article qualifies for 1 CE credit.

Appendix A. Econometric Model

The returns to a given asset can be expressed as a linear combination of risk factor returns, as shown in Equation 1:

$$r_t = \alpha + \sum_i \beta_i f_{i,t} + \epsilon_t, \quad (1)$$

where

- r_t = the return of the asset
- α_t = the intercept
- β_i = the exposure of the asset to the i th factor
- f_i = return for the i th factor
- ϵ_t = an error term

To derive our econometric approach, we assumed that the observed “smoothed” returns for each illiquid asset can be viewed as a weighted average of the recent history of actual, but unobserved, returns, as shown in Equation 2:

$$r_{obs,t} = \sum_j \omega_j r_{t-j}, \quad (2)$$

where

- $r_{obs,t}$ = the observed index return
- Q = the number of lags
- r_t = the unobserved actual investment return
- ω_j = weights that reflect how past realized investment returns affect the current observed, smoothed return⁸

Essentially, this model assumes that the observed return series, r_{obs} , can be viewed as a so-called moving-average process of past investment returns, r , with normalized coefficients equal to $\{\omega_j\}$.⁹ Therefore, the observed index return can be written as a function of past risk factor returns, as shown in Equation 3:

$$\begin{aligned} r_{obs,t} &= \sum_j \omega_j \left(\alpha + \sum_i \beta_i f_{i,t-j} + \epsilon_{t-j} \right) \\ &= \sum_j \omega_j \alpha + \sum_i \beta_i \sum_j \omega_j f_{i,t-j} + \sum_j \omega_j \epsilon_{t-j}, \end{aligned} \quad (3)$$

where N is the number of risk factors. If we define $X_{i,t} = \sum_j \omega_j f_{i,t-j}$ as the transformed (“moving-average”) risk factor returns and $\eta_t = \sum_j \omega_j \epsilon_{t-j}$ as the weighted error term, it then follows that we can estimate risk factor betas (β_i) on $X_{i,t}$ directly, as shown in Equation 4:

$$r_{obs,t} = \alpha + \sum_i \beta_i X_{i,t} + \eta_t. \quad (4)$$

The parameters of this joint model of actual and smoothed illiquid asset returns can be estimated in two steps. The lag weights $\{\omega_j\}$ are first

estimated with maximum likelihood on observed (“smoothed”) asset returns. For each asset, an appropriate number of lags is selected on the basis of their statistical significance.

In the second step, these estimates for $\{\omega_i\}$ are used to construct the appropriately weighted factor return time series $\{X_{i,t}\}$. The factor loadings, β_i , are then estimated from Equation 1 using ordinary least squares. Because the error terms, $\eta_{i,t}$, will be autocorrelated, we use Newey–West corrected standard errors to assess statistical significance for each estimated factor exposure.

Appendix B. Data Sources

In this appendix, we provide the data sources for risk factor returns and asset-class returns.

Risk Factor Returns

Equity: S&P 500 Index; DataStream

Size, value, leverage: Barra GEM2 style factor returns

Industry factors: Barra GEM2 industry factor returns

Duration: US government 10-year yield; DataStream

Real duration: US government 10-year yield minus 10-year inflation expectations (Livingston Survey); DataStream

Corporate spread: Barclays US Aggregate Credit average option-adjusted spread; Bloomberg

High-yield spread: Barclays US High Yield average option-adjusted spread; Bloomberg

EM spread: J.P. Morgan Emerging Market Bond Index Global Composite (return per unit of spread duration); DataStream; PIMCO

Commodity: Dow Jones-UBS Commodity Index spot; DataStream

Liquidity: Pastor–Stambaugh liquidity factor (Lubos Pastor’s University of Chicago Booth School of Business web page: <http://faculty.chicagobooth.edu/lubos.pastor/research/>); PIMCO

Volatility: CBOE Volatility Index; Bloomberg

Momentum: AQR Momentum Total Return Index; Bloomberg

Asset-Class Returns

Private equity: Cambridge Associates U.S. Private Equity Index

Venture capital: Cambridge Associates U.S. Venture Capital Index

Infrastructure: UBS Global Infrastructure and Utilities (listed); Bloomberg

Farmland: NCREIF Farmland Index

Timberland: NCREIF Timberland Index

Real estate: NCREIF Property, Core, Value Added, and Opportunistic indices

Hedge funds: HFRI Composite Index; Bloomberg Equities: S&P 100 Index; DataStream

Government bonds: Barclays US Government Bond Index; Bloomberg

Notes

- For example, Pedersen, He, Tiwari, and Hoffmann (2012) showed that private real estate’s risk characteristics closely resemble those of public real estate after the private return series have been appropriately adjusted for “appraisal” biases (see also Fisher, Geltner, and Webb 1994; Fisher and Geltner 2000). Asness, Krail, and Liew (2001) showed that the same principle applies to hedge fund returns.
- As an illustration of the pitfalls of this approach, Leinweber (2007) used a large set of economic data to find factors that fitted the S&P 500 Index. He showed that a model that combined (1) butter production in Bangladesh, (2) cheese production in the United States, and (3) the sheep population in Bangladesh explained the returns of the S&P 500 with an R^2 of 99%—a clearly spurious result due to overfitting.
- Note that these betas may be unstable across market regimes. Hence, the exposures we report here should be adjusted on the basis of the market environment. An easy way to adjust betas to the current market environment is to focus on past data that represent similar conditions—for example, by shortening the regression window to include only recent data or by using data from a specific type of regime, such as “turbulent” or “rising rates” regimes. Absent a view on the market regime, we recommend using an unconditional estimate based on data that cover multiple business cycles, which is what we did in our study. Doing so will help “identify” a robust set of risk factor exposures for the individual assets.
- For example, a recession may reduce demand for office and retail space, which, in turn, negatively affects the occupancy rates and net operating income of commercial real estate properties. Therefore, changes in prospective equity market earnings should also be positively correlated with changes in projected cash flows from private investments.
- For hedge fund risk analysis and manager selection, it is particularly important to complement the risk factor approach with a due diligence process that provides a more holistic view of individual managers’ activities. Mapping an individual hedge fund to risk factors on the basis of its hedge fund style category may be ill-advised because, as mentioned, hedge funds often deviate substantially from hedge funds in the same category or from the average fund in their category. Some managers may also be selling or buying options, which gives rise to nonlinear factor exposures that become evident only in tail events and crisis episodes. These exposures can be difficult to identify during periods when financial markets are well behaved. Access to short-term funding is important to most hedge funds because they rely on significant leverage to achieve their investment objectives or to implement relative value strategies. Also, we do not explicitly address the risk associated with “forced” deleveraging during financial crises in our risk factor models, but this risk is an important dimension of the overall “tail” risk for hedge funds. Finally, note that there is potential for direct or indirect contagion across hedge funds owing to the complex and

illiquid nature of the funds' activities. Such joint dependencies are extremely challenging to model and are beyond the scope of our risk factor analysis.

6. As a caveat, although Table 1 presents relatively long-run estimates, econometric methodologies that account for time-varying betas—such as a dynamic conditional correlation GARCH model (Engle 2002)—may help enhance the approach presented here.
7. A few of the risk factors themselves are serially correlated owing to valuation and liquidity effects. In particular, *F*-tests suggest significant smoothing for EM equity, credit spread, value, leverage, and the software and transportation industry factors.
8. Formally, the weights are assumed/normalized to satisfy the

following conditions: $\sum_j^Q \omega_j = 1$, and $\omega_j > 0$

9. The specification implies an MA(*q*) process for returns. This approach is based on the additional assumption that actual returns are identically and independently distributed over time. The parameters of the MA(*q*) process can be estimated using standard software packages. We used the ARMAX filter function in MATLAB (from Kevin Sheppard's Econometrics Toolbox, available at www.kevinsheppard.com).

The estimation process also gives an estimate of the actual unsmoothed investment returns. In general, one might ask whether a simple ordinary least-squares (OLS) regression would lead to similar factor betas when compared with our approach (for a related discussion on hedge fund returns, see Edelman, Fung, and Hsieh 2013). Under specific circumstances, conventional OLS-based multiple regressions may yield exposures similar to those from our autoregressive model. In a separate experiment, we repeated all regressions shown in Figure 2 but removed the lag structure described in Equation 3. We found that across alternative asset classes and strategies, all equity, liquidity, duration, and spread betas were lower without the adjustment. Overall, betas went down by an average of 64% of their initial values. However, statistical significance decreased; 66% of the loadings had *t*-statistics that decreased by 0.5 or more, and 28% had *t*-statistics that decreased by 2.0 or more. The most significant differences in factor betas were for real estate (across unlevered, core, value added, and opportunistic), but farmland, timberland, private equity, and venture capital also had meaningful differences. Public markets did not have any meaningful difference, as expected. These results are consistent with the significance of the differences in volatility estimates, as shown by the *F*-tests in Table 3.

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